BACKWARDS MATH

Suggested Grades

9^{th} or 10^{th} Grade Algebra

SD Mathematics Strand & Standard *(Primary for Task)*

Algebra

9-12.A.2.1. Students are able to use algebraic properties to transform multi-step, single-variable, and first-degree equations.

9-12.A.2.2. Students are able to use algebraic properties to transform multi-step, single-variable, and first-degree inequalities and represent solutions using a number line.

Task Summary

Students develop and justify equation or inequality problems for given solutions.

Time and Context of Task

1-2 class periods

Individual or student pairs

Materials Needed

Backwards Math task and Solution worksheet; Paper, Pencil, Calculator

Author and Lead Teacher for This Task

Cindy Kroon

Montrose High School

BACKWARDS MATH

You will be supplied with the answers to 20 algebra problems. Your task is to create an equation or inequality problem for each given solution.

- Problems must use a single variable, although the variable may appear more than once in a given problem.
- All equations/inequalities must be first degree.
- Only integers may be used as coefficients.
- Problems must be multi-step (include at least 4 problems with 3 or more steps).
- Organize your problems in worksheet format.
- Provide a separate page with worked-out solutions and explanations for each problem.

Trade worksheets with another individual or team. Solve the problems and check work for accuracy. Provide feedback. Consider the feedback provided to you. Revise your work accordingly.

Example:

Answer: x = 2
Problem: 5x + 7 = 13 + 2x

Explanation: (on separate sheet)

5x + 7 = 13 + 2x	
-2x -2x	(regroup variable)
3x + 7 = 13	
-7 -7	(combine units)
3x = 6	
x = 2	(divide to solve)

Backwards Math Solutions

Directions:
Create an equation or inequality problem that will result in the solution given. Check that each equation or inequality meets the conditions listed in the task.

1) $x = 3$
2) $n = -2$
3) $q = 0$
4) $y = 4$
5) $p = -7$
6) $z = 5$
7) $x = 2.5$
8) $m = 1$
9) $r = -3$
10) $z = -1.5$
11) $x > 4$
12) $w < 2$
13) $p \geq -3$
14) $z < 0$
15) $q \leq 1$
16) $n > -4$
17) $y \geq -1$
18) $z < -5$
19) $x < -2.5$
20) $n \leq 3$

CONTENT STANDARDS

Primary Standards

Strand Name:	**Algebra**
SD Goal 1:	Students will use the language of algebra to explore, describe, represent, and analyze number expressions and relations that represent variable quantities.
Indicator 2:	Use a variety of algebraic concepts and methods to solve equations and inequalities**.**
Standard:	9-12.A.2.1. Students are able to use algebraic properties to transform multi-step, single-variable, and first-degree equations.
Standard	9-12.A.2.2. Students are able to use algebraic properties to transform multi-step, single-variable, and first-degree inequalities and represent solutions using a number line.

NCTM Process Standards

Problem Solving:	Apply and adapt a variety of appropriate strategies to solve problems. Monitor and reflect on the process of mathematical problem solving.
Representation:	Select, apply and translate among mathematical representations to solve problems. Create and use representations to organize, record, and communicate mathematical ideas.
Communication:	Use the language of mathematics to express mathematical ideas precisely.

Problem-Solving Strategies

- Estimation and check
- Developing formulas and writing equations
- Working backward
- Insufficient information
- Looking for patterns

ASSESSMENT TOOLS

Task Rubric

CATEGORY	Advanced	Proficient	Basic	Below Basic
SD Standard: 9-12.A.2.1. Students are able to use algebraic properties to transform multi-step, single-variable, and first-degree equations.	Explanation shows complete understanding of the mathematical concepts used to solve the problem(s). Problems are 3 or more steps.	Explanation shows substantial understanding of the mathematical concepts used to solve the problem(s). Problems are 2-step.	Explanation shows some understanding of the mathematical concepts needed to solve the problem(s). Some problems are not 2-step or are not solved correctly.	Explanation shows very limited understanding of the underlying concepts needed to solve the problem(s) OR is not written.
SD Standard: 9-12.A.2.2. Students are able to use algebraic properties to transform multi-step, single-variable, and first-degree inequalities and represent solutions using a number line.	Explanation shows complete understanding of the mathematical concepts used to solve the problem(s). Problems are 3 or more steps.	Explanation shows substantial understanding of the mathematical concepts used to solve the problem(s). Problems are 2-step.	Explanation shows some understanding of the mathematical concepts needed to solve the problem(s). Some problems are not 2-step or are not solved correctly.	Explanation shows very limited understanding of the underlying concepts needed to solve the problem(s) OR is not written.
NCTM Process Standard Communication: Use the language of mathematics to express mathematical ideas precisely.	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminology and notation are used, but it is sometimes not easy to understand what was done.	There is little use, or a lot of inappropriate use, of terminology and notation.
NCTM Process Standard Problem Solving: Apply and adapt a variety of appropriate strategies to solve problems.	Typically, uses an efficient and effective strategy to solve the problem(s).	Typically, uses an effective strategy to solve the problem(s).	Sometimes uses an effective strategy to solve problems, but does not do it consistently.	Rarely uses an effective strategy to solve problems.
NCTM Process Standard Problem Solving: Monitor and reflect on the process of mathematical problem solving	Uses complex and refined mathematical reasoning.	Uses effective mathematical reasoning	Some evidence of mathematical reasoning.	Little evidence of mathematical reasoning.

Rubric created using Rubistar http://rubistar.4teachers.org/index.php

Core High School Algebra
Performance Descriptors

Advanced	**High school students performing at the advanced level:** • transform algebraic expressions; • solve quadratic equations; • solve a system of linear equations.
Proficient	**High school students performing at the proficient level:** • transform polynomial expressions using real number properties; • solve single variable linear equations with integral coefficients; • graph linear equations; • interpret tables, graphs, and charts to solve problems; • create a linear model from a problem context.
Basic	**High school students performing at the basic level:** • transform linear expressions with integral coefficients using real number properties; • solve linear equations of the form $ax + b = c$, where a, b, and c are integers; • recognize the graph of a linear equation; • graph a line from a table of values.

Core High School Algebra
ELL Performance Descriptors

Proficient	**High school ELL students performing at the proficient level:** • solve, transform, and graph linear equations; • apply algebraic representations to solve problems; • read, write, and speak the language of algebra and apply it to algebraic problem-solving situations.
Intermediate	**High school ELL students performing at the intermediate level:** • solve one-variable linear equations; • graph linear equations in slope-intercept form; • complete tables to graph linear equations; • create numerical expressions from oral or written contexts; • evaluate an algebraic expression given the value of the variable(s); • explain in algebraic terms the steps and/or strategies used in problem solving; • give oral, pictorial, symbolic (diagrams) or written responses to questions on topics presented in class.
Basic	**High school ELL students performing at the basic level:** • graph points on a coordinate system; • solve problems with integral and rational solutions; • evaluate numerical expressions; • demonstrate problem-solving strategies; • break tasks into smaller parts and make connections to prior knowledge; • recognize, compare, and use appropriate algebraic terms; • respond to yes or no questions and to problems presented pictorially or numerically in class.
Emergent	**High school ELL students performing at the emergent level:** • identify and use mathematical symbols; • copy and write numerals and algebraic symbols; • imitate pronunciation of numerals and mathematical terms; • use non-verbal communication to express mathematical ideas.
Pre-emergent	**High school ELL students performing at the pre-emergent level:** • observe and model appropriate cultural and learning behaviors from peers and adults; • listen to and observe comprehensible instruction and communicate understanding non-verbally.

BACKWARDS MATH

Student Work Samples

As you examine the samples, consider the following questions:

- In light of the standard/s addressed and the assessment tools provided, what evidence does the work provide that students are achieving proficiency in the knowledge and skills addressed by the standard/s for the task?
- Is the task/activity well designed to help students acquire knowledge and demonstrate proficiency? Is the task/activity clearly aligned with the standards? In what ways would you adapt the task/activity to better meet the needs of your students?

Student Work Sample #1
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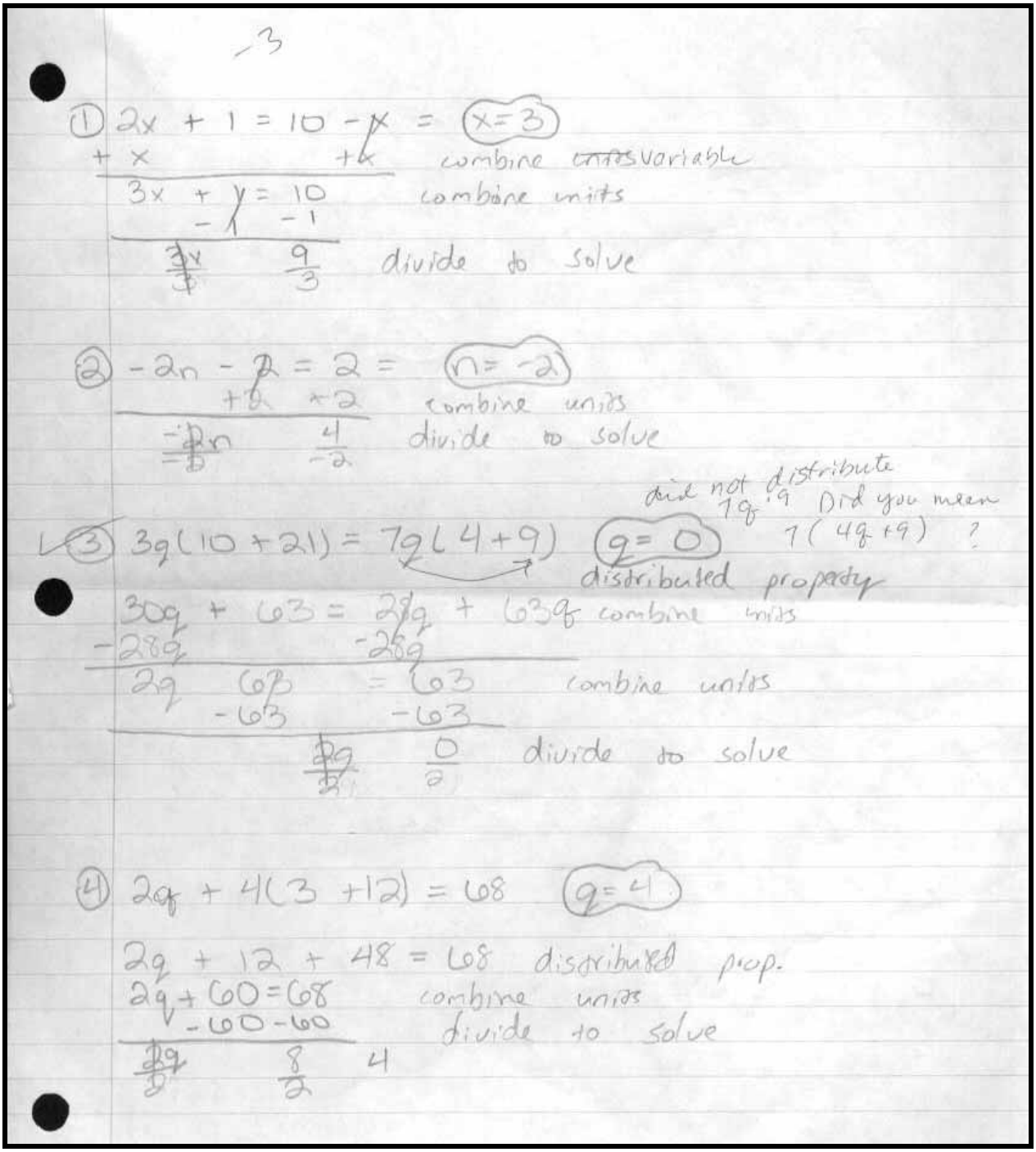
1. $2x+1=10-x$
2. $-2n-2=2$
3. $3q(10+21)=7q(4+9)$
4. $2y+4(3+12)=68$
5. $2p+7=-7$
6. $5z-10=3(5)-3+3$
7. $2x+10=15$
8. $8m-32=4m(12-18)$
9. $6r+18=-3(0)$
10. $4z+10=2z+7$
11. $2x+6>14$
12. $3w+6<12$
13. $2p+10\geq 1p+7$
14. $3z(10+490)<7z(4+210)$
15. $3q+6\leq 9$
16. $94n-8(6+24)> 136$
17. $3y+15\geq 12$
18. $3z+6<-9$
19. $2x+25<20$
20. $4n+2\leq 20-2n$



-3

① $2x + 1 = 10 - x = x = 3$

$+x \quad +x$ combine variable

$3x + 1 = 10$ combine units

$-1 \quad -1$

$\frac{3x}{3} \quad \frac{9}{3}$ divide to solve

② $-2n - 2 = 2 = n = -2$

$+2 \quad +2$ combine units

$\frac{-2n}{-2} \quad \frac{4}{-2}$ divide to solve

did not distribute 7q ? Did you mean 7(4q+9) ?

③ $3q(10+21) = 7q(4+9)$ $q = 0$

distributed property

$30q + 63 = 28q + 63q$ combine units

$-28q \quad -28q$

$2q + 63 = 63$ combine units

$-63 \quad -63$

$\frac{2q}{2} \quad \frac{0}{2}$ divide to solve

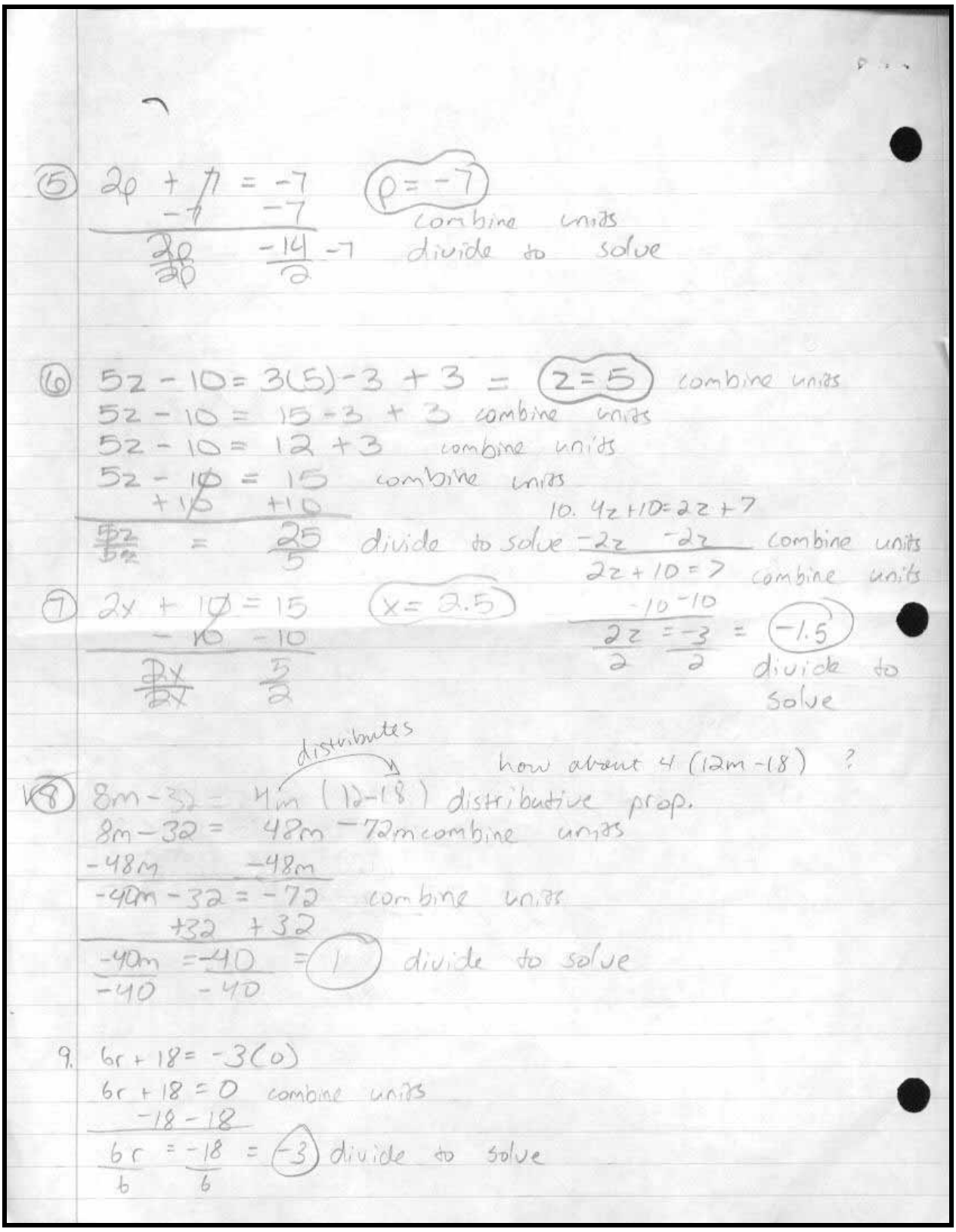
④ $2q + 4(3+12) = 68$ $q = 4$

$2q + 12 + 48 = 68$ distributed prop.

$2q + 60 = 68$ combine units

$-60 \quad -60$ divide to solve

$\frac{2q}{2} \quad \frac{8}{2} \quad 4$



5. $2p + 7 = -7$ $(p = -7)$
$-7 \quad -7$ combine units
$\frac{2p}{2} = \frac{-14}{2}$ divide to solve

6. $5z - 10 = 3(5) - 3 + 3 = (z = 5)$ combine units
$5z - 10 = 15 - 3 + 3$ combine units
$5z - 10 = 12 + 3$ combine units
$5z - 10 = 15$ combine units
$+10 \quad +10$
$\frac{5z}{5} = \frac{25}{5}$ divide to solve

7. $2x + 10 = 15$ $(x = 2.5)$
$-10 \quad -10$
$\frac{2x}{2} = \frac{5}{2}$

10. $4z + 10 = 2z + 7$
$-2z \quad -2z$ combine units
$2z + 10 = 7$ combine units
$-10 \quad -10$
$\frac{2z}{2} = \frac{-3}{2} = (-1.5)$ divide to solve

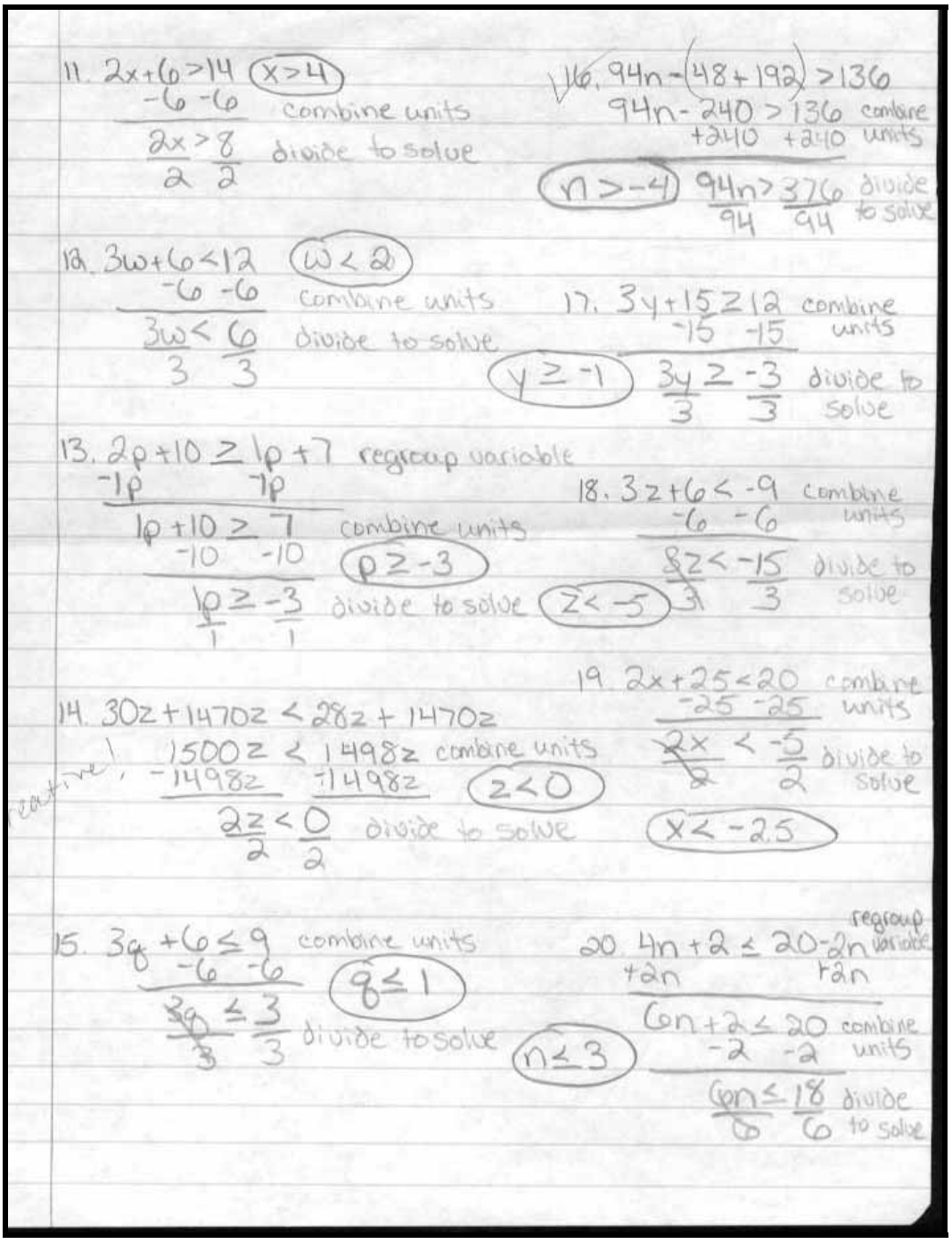
distributes

how about $4(12m - 18)$?

8. $8m - 32 = 4m(12 - 18)$ distributive prop.
$8m - 32 = 48m - 72m$ combine units
$-48m \quad -48m$
$-40m - 32 = -72$ combine units
$+32 \quad +32$
$\frac{-40m}{-40} = \frac{-40}{-40} = (1)$ divide to solve

9. $6r + 18 = -3(0)$
$6r + 18 = 0$ combine units
$-18 \quad -18$
$\frac{6r}{6} = \frac{-18}{6} = (-3)$ divide to solve

Sample #1 – page 4 of 4



Looking at Student Work – Instructor notes and rating for work sample #1:

This project was rated as proficient. The students' explanation shows substantial understanding of the mathematical concepts used to solve the problems. Many problems are 3 or more steps. A few errors with the distributive property prevented this project from being considered advanced.

Correct terminology and notation are consistently used making it easy to understand what was done. Students used an effective strategy to solve the problem, and demonstrated effective mathematical reasoning.

Student Work Sample #2
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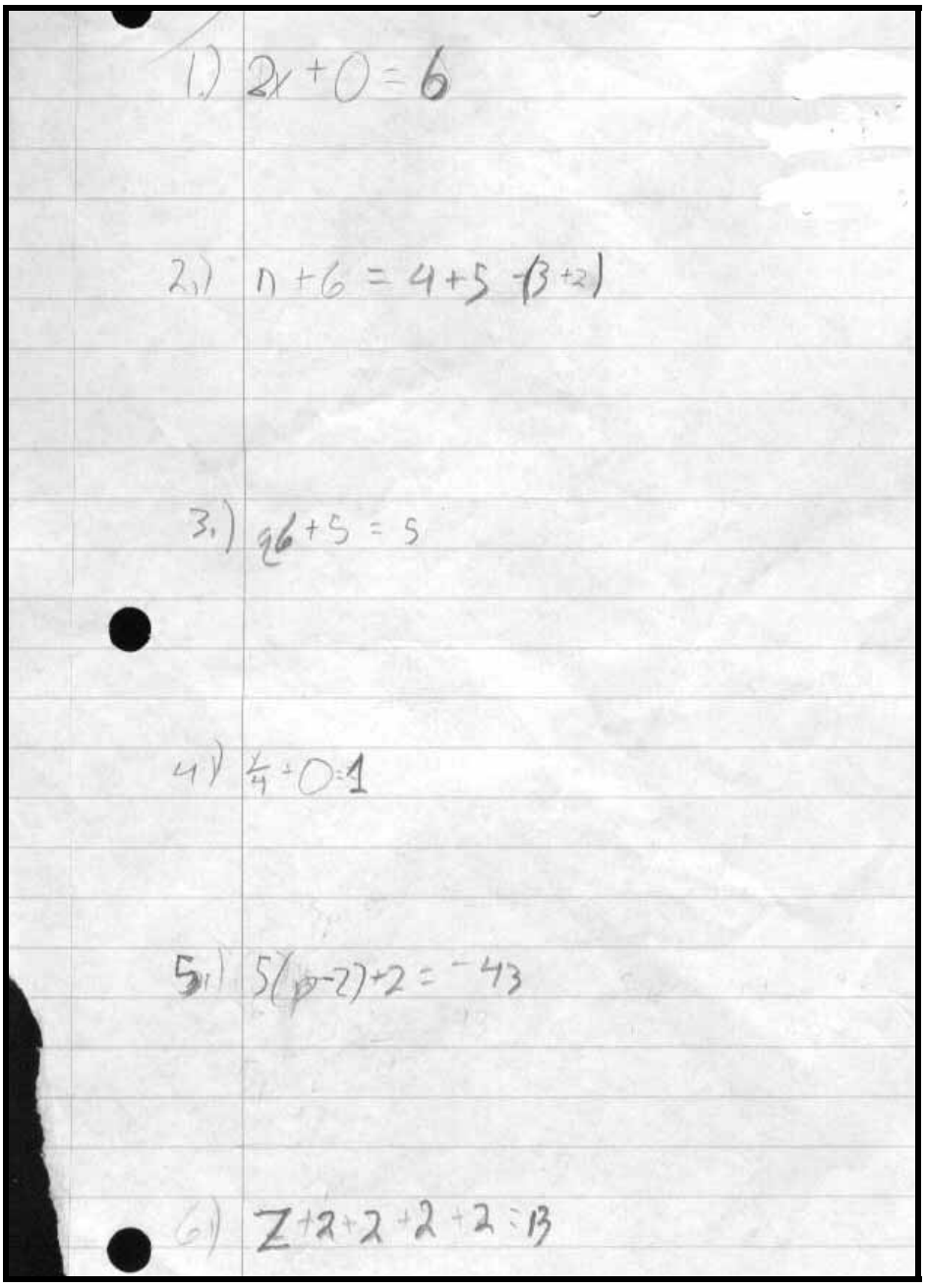
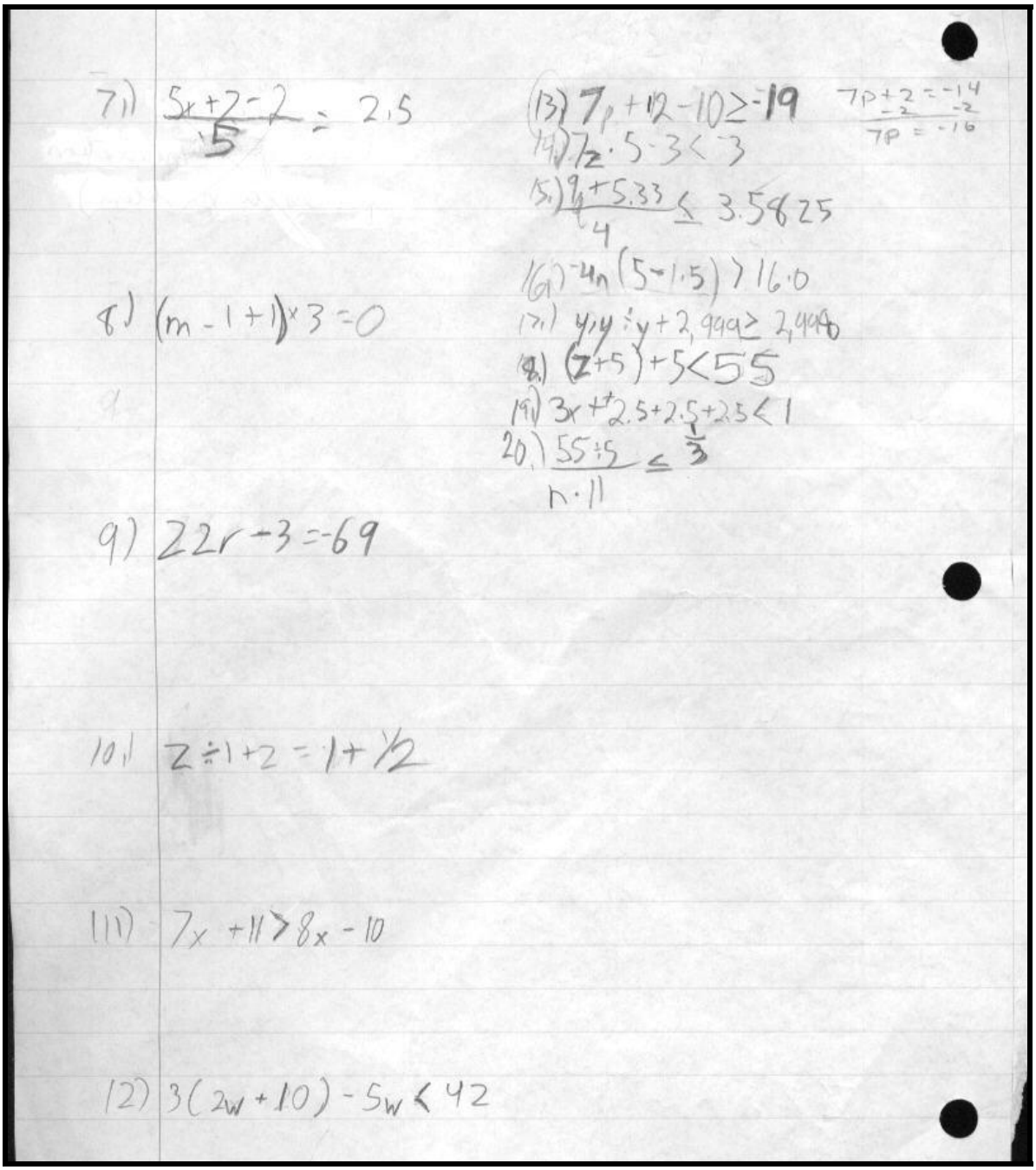
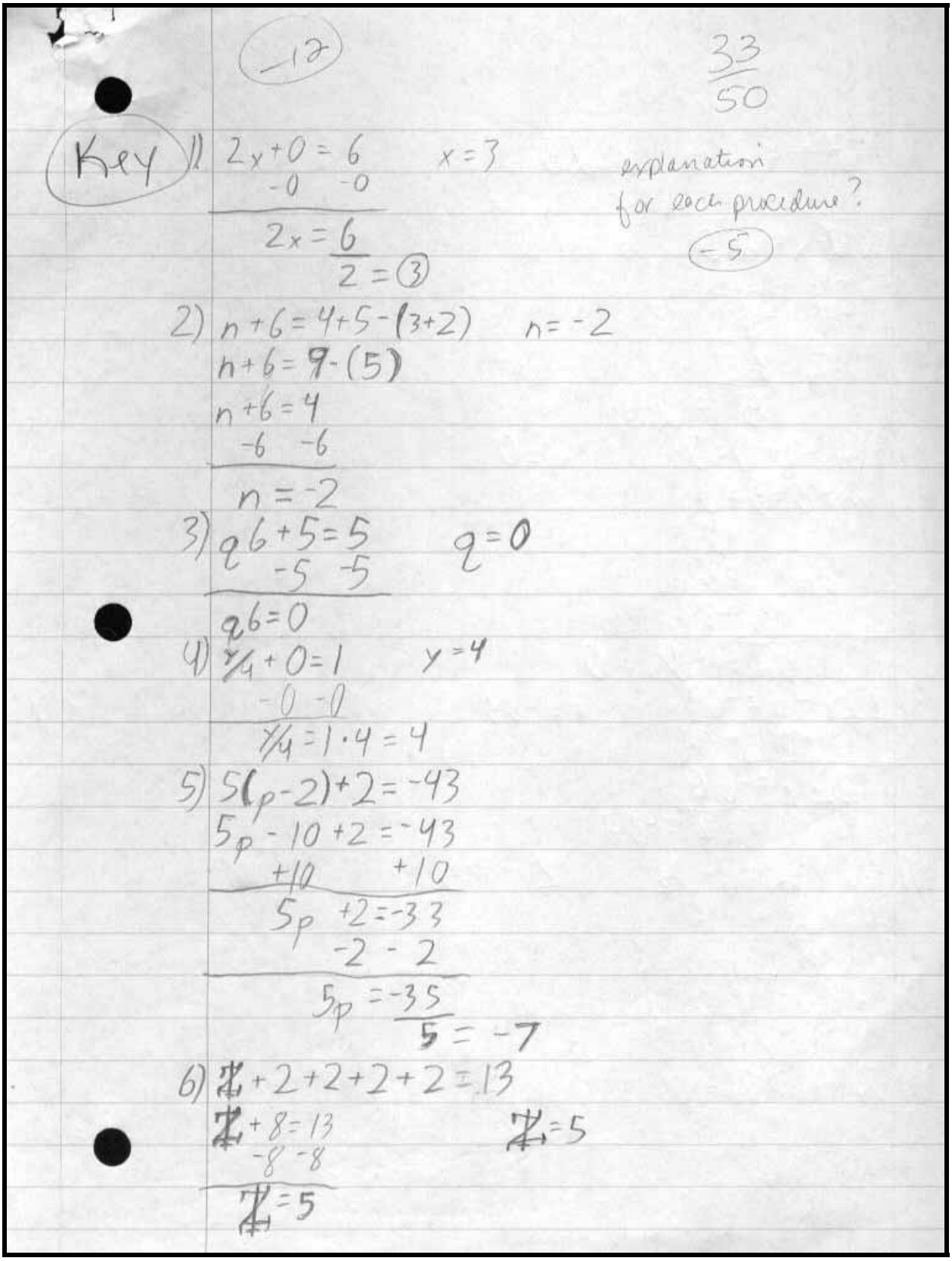



7) $\frac{5x+2-2}{5} = 2.5$
8) $(m-1+1)\times 3 = 0$
9) $22r-3=-69$
10) $2\div 1+2 = 1+\frac{1}{2}$
11) $7x+11 > 8x-10$
12) $3(2w+10)-5w < 42$
13) $7p+12-10 \geq -19$
$7p+2=-14$
$-2 \quad -2$
$7p=-16$
14) $7z \cdot 5-3 < 3$
15) $\frac{9}{4}+5.33 \leq 3.5 \& 25$
16) $-4n(5-1.5) > 16.0$
17) $y,y \div y+2,999 \geq 2,000$
18) $(z+5)+5<55$
19) $3x+2.5+2.5+2.5<1$
20) $\frac{55\div 5}{n \cdot 11} \leq \frac{1}{3}$



-12

33/50

Key 1) $2x+0=6$ $x=3$
$-0 \quad -0$
$2x=6$
$\frac{2x}{2}=\frac{6}{2}=3$

explanation for each procedure?

-5

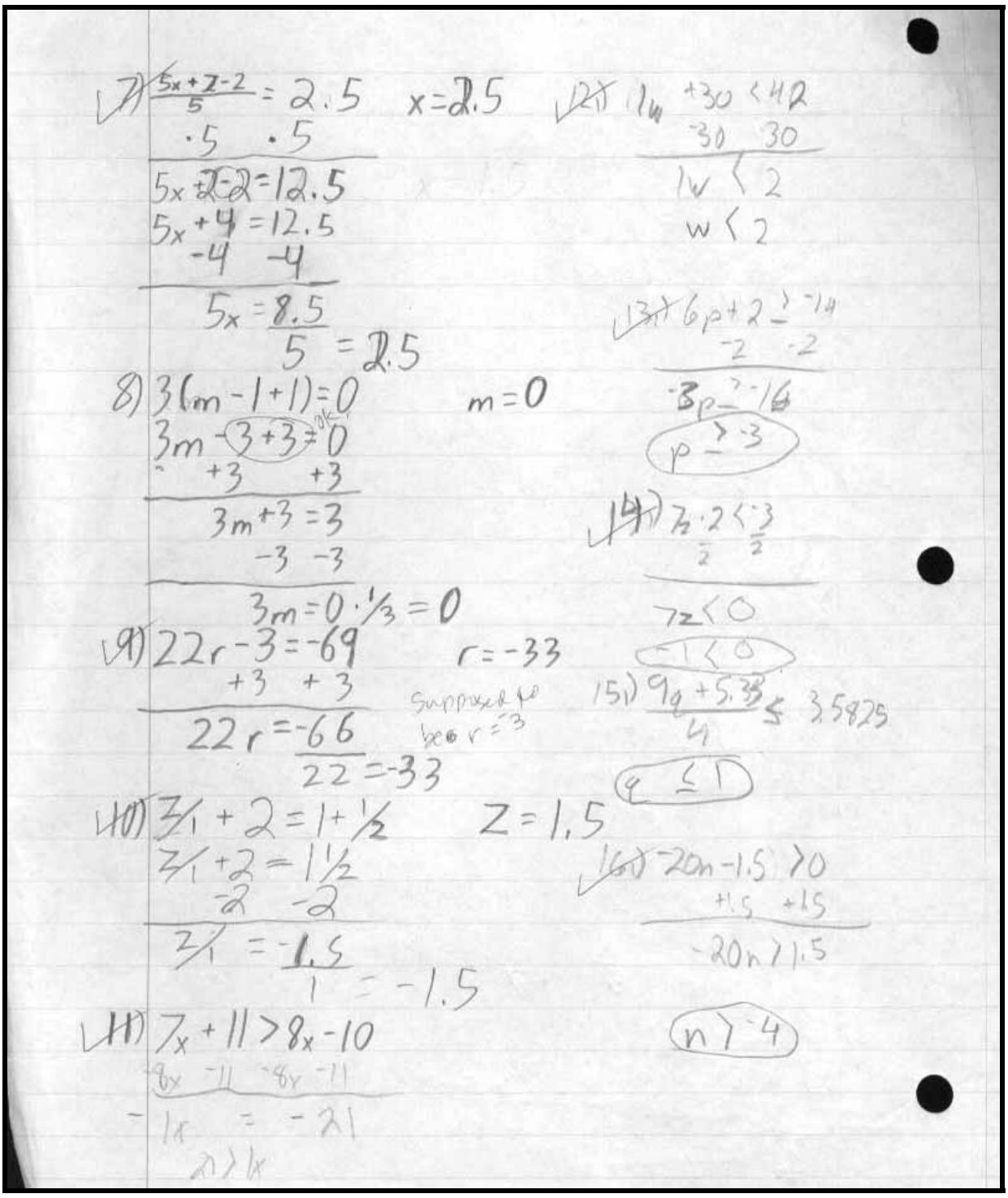
2) $n+6=4+5-(3+2)$ $n=-2$
$n+6=9-(5)$
$n+6=4$
$-6 \quad -6$
$n=-2$

3) $q6+5=5$ $q=0$
$-5 \quad -5$
$q6=0$

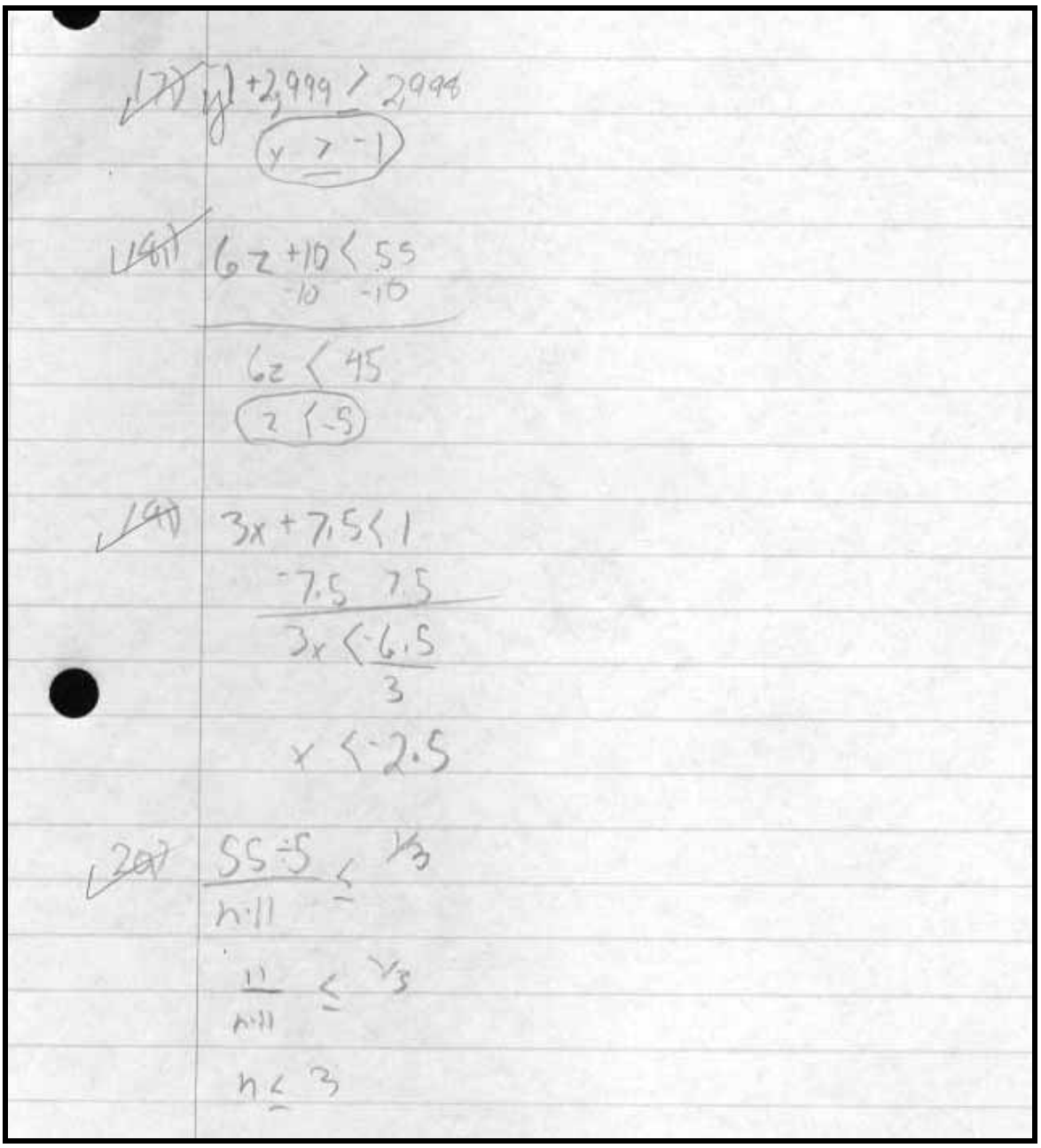
4) $\frac{y}{4}+0=1$ $y=4$
$-0 \quad -0$
$\frac{y}{4}=1\cdot 4=4$

5) $5(p-2)+2=-43$
$5p-10+2=-43$
$+10 \quad +10$
$5p+2=-33$
$-2 \quad -2$
$5p=-35$
$\frac{5p}{5}=-7$

6) $x+2+2+2+2=13$
$x+8=13$ $x=5$
$-8 \quad -8$
$x=5$


7) (5x+7-2)/5 = 2.5 x=2.5
·5 ·5
5x+7-2=12.5
5x+4=12.5
-4 -4
5x=8.5
5 = 2.5
8) 3(m-1+1)=0 m=0
3m-3+3=0
+3 +3
3m+3=3
-3 -3
3m=0 ·1/3=0
9) 22r-3=-69 r=-33
+3 +3
22r=-66
22=-33
Supposed to be r=-3
10) 3/1 + 2 = 1+1/2 z=1.5
3/1 +2=1 1/2
-2 -2
3/1 = -1.5
1 = -1.5
11) 7x+11>8x-10
-8x -11 -8x -11
-1x = -21
x>1x
12) 14 +30 < 42
-30 30
1w < 2
w < 2
13) 6p+2 ≥ -14
-2 -2
-8p ≥ -16
p ≥ -3
14) 7/2 · 2 < 3/2
7z < 0
-1 < 0
15) 9q+5.35/4 ≤ 3.5875
q ≤ 1
16) -20n-1.5 > 0
+1.5 +1.5
-20n > 1.5
n > -4

Sample #2 – page 5 of 5



Looking at Student Work – Instructor notes and rating for work sample #2:

This project was assigned a rating of basic. The student explanation shows some understanding of the mathematical concepts for solving equalities, but very limited understanding of inequalities. Although correct terminology and notation are usually used, an effective strategy is not consistently employed. There is some evidence of mathematical reasoning but the students have not demonstrated mastery of the concepts.

INSTRUCTIONAL NOTES

Task Extensions

- Allow/require rational numbers in student equations/inequalities.
- Adjust difficulty by using rational number solutions or whole numbers only.
- Specify a minimum of 3-step solutions.
- Allow one-step solutions.
- Allow second degree or higher equations.
- Since students take more seriously what is evaluated, I anticipate that the project evaluation (point grade) will count approximately ½ that of a test grade.

Resources

SD Mathematics Content Standards
http://www.doe.sd.gov/contentstandards/math/index.asp
SD Assessment and Testing
http://www.doe.sd.gov/octa/assessment/index.asp
The National Assessment of Educational Progress (NAEP)
http://www.doe.sd.gov/octa/assessment/naep/index.asp
National Council of Teachers of Mathematics
http://nctm.org/
Looking at Student Work
http://www.lasw.org/index.html